1. (1 pt) 
Use the Integral Test to determine whether the infinite series is convergent.

\[ \sum_{n=1}^{\infty} n^{-\frac{1}{6}} \]

Fill in the corresponding integrand and the value of the improper integral.

Enter inf for \( \infty \), -inf for \(-\infty\), and DNE if the limit does not exist.

Compare with \( \int_{1}^{\infty} \) \( x^{-\frac{1}{6}} \) \( dx \) =

By the Integral Test, the infinite series \( \sum_{n=3}^{\infty} \frac{1}{n+25} \)

- A. converges
- B. diverges

SOLUTION:

Let \( f(x) = x^{-\frac{1}{6}} \). This function is continuous, positive and decreasing on the interval \( x \geq 1 \), so the Integral Test applies.

Moreover,

\[ \int_{1}^{\infty} x^{-\frac{1}{6}} dx = \lim_{R \to \infty} \int_{1}^{R} x^{-\frac{1}{6}} dx = \frac{6}{5} \lim_{R \to \infty} \left( R^{\frac{5}{6}} - 1 \right) = \infty. \]

The integral diverges; hence the series \( \sum_{n=1}^{\infty} n^{-\frac{1}{6}} \) also diverges.

Answer(s) submitted:
- \(-1/6\)
- inf
- B (correct)

Correct Answers:
- \(-1/6\)
- inf
- B

2. (1 pt)
Use the Integral Test to determine whether the infinite series is convergent.

\[ \sum_{n=1}^{\infty} \frac{1}{n+25} \]

Fill in the corresponding integrand and the value of the improper integral.

Enter inf for \( \infty \), -inf for \(-\infty\), and DNE if the limit does not exist.

Compare with \( \int_{5}^{\infty} \) \( \frac{1}{x+25} \) \( dx \) =

By the Integral Test, the infinite series \( \sum_{n=1}^{\infty} \frac{1}{n+25} \)

- A. converges
- B. diverges

SOLUTION:

Let \( f(x) = \frac{1}{x+25} \). This function is continuous, positive and decreasing on the interval \( x \geq 5 \), so the Integral Test applies.

Moreover,

\[ \int_{5}^{\infty} \frac{1}{x+25} dx = \lim_{R \to \infty} \int_{5}^{R} \frac{1}{x+25} dx = \lim_{R \to \infty} \left( \ln(R+25) - \ln(30) \right) = \infty. \]

The integral diverges; hence the series \( \sum_{n=5}^{\infty} \frac{1}{n+25} \) also diverges.

Answer(s) submitted:
- \(-1/6, 0.166667\)
- inf
- B (correct)

Correct Answers:
- \(-1/6, 0.166667\)
- inf
- B

3. (1 pt)
Use the Integral Test to determine whether the infinite series is convergent.

\[ \sum_{n=2}^{\infty} \frac{8\ln n}{n^2} \]

Fill in the corresponding integrand and the value of the improper integral.

Enter inf for \( \infty \), -inf for \(-\infty\), and DNE if the limit does not exist.

Compare with \( \int_{2}^{\infty} \) \( \frac{8\ln x}{x^2} \) \( dx \) =

By the Integral Test, the infinite series \( \sum_{n=2}^{\infty} \frac{8\ln n}{n^2} \)

- A. converges
- B. diverges

SOLUTION:

Let \( f(x) = \frac{8\ln x}{x^2} \). Because
\[ f'(x) = \frac{8(1 - 2\ln x)}{x^3}, \]

we see that \( f'(x) > 0 \) for \( x > \sqrt{e} \approx 1.65 \). We conclude that \( f \) is decreasing on the interval \( x \geq 2 \). Since \( f \) is also positive and continuous on this interval, the Integral Test can be applied. By Integration by Parts we find

\[
\int_2^\infty \frac{8\ln x}{x^2} \, dx = -\frac{\ln x}{x} \bigg|_2^\infty + \int_2^\infty \frac{-1}{x} \, dx = -\ln x \bigg|_2^\infty + C;
\]

therefore,

\[
\int_2^\infty \frac{8\ln x}{x^2} \, dx = \lim_{R \to \infty} \int_2^R \frac{8\ln x}{x^2} \, dx = 8 \lim_{R \to \infty} \left( \frac{1}{2} + \frac{\ln 2}{2} - \frac{1}{R} - \frac{\ln R}{R} \right) + \left( -\ln 2 \right).
\]

We compute the resulting limit using L'Hopital's Rule:

\[
\lim_{R \to \infty} \frac{\ln R}{R} = \lim_{R \to \infty} \frac{1}{R} = 0.
\]

Hence,

\[
\int_2^\infty \frac{8\ln x}{x^2} \, dx = \frac{8(1 + \ln 2)}{2}.
\]

The integral converges; therefore, the series \( \sum_{n=2}^\infty \frac{8\ln n}{n^2} \) also converges.

Answer(s) submitted:
- 8ln/x/x^2
- 4(ln2+1)
- A

(correct)
Correct Answers:
- 8*ln(x)/x^2
- 6.77259
- A

4. (1 pt)

Use the Comparison Test to determine whether the infinite series is convergent.

\( \sum_{n=1}^\infty \frac{1}{n9^n} \)

By the Comparison Test, the infinite series \( \sum_{n=1}^\infty \frac{1}{n9^n} \)

- A. converges
- B. diverges

Note: You are allowed only one attempt on this problem.

SOLUTION:

Solution:

We compare with the geometric series \( \sum \left( \frac{1}{9} \right)^n \). For \( n \geq 1 \),

\[
\frac{1}{n9^n} \leq \frac{1}{9^n} = \left( \frac{1}{9} \right)^n.
\]

Since \( \sum_{n=1}^\infty \left( \frac{1}{9} \right)^n \) converges (it's a geometric series with \( |r| = \frac{1}{9} < 1 \)), we conclude by the Comparison Test that \( \sum_{n=1}^\infty \frac{1}{n9^n} \) also converges.

Answer(s) submitted:
- A

(correct)
Correct Answers:
- A

5. (1 pt)

Use the Comparison Test to determine whether the infinite series is convergent.

\( \sum_{n=1}^\infty \frac{\sin^6 n}{n^8} \)

By the Comparison Test, the infinite series \( \sum_{n=1}^\infty \frac{\sin^6 n}{n^8} \)

- A. converges
- B. diverges

Note: You are allowed only one attempt on this problem.

SOLUTION:

Solution:

For \( n \geq 1 \), \( 0 \leq \sin^6 n \leq 1 \), so

\[
0 \leq \frac{\sin^6 n}{n^8} \leq \frac{1}{n^8}.
\]

The series \( \sum_{n=1}^\infty \frac{1}{n^8} \) is a p-series with \( p = 8 > 1 \), so it converges.

By the Comparison Test we can therefore conclude that series \( \sum_{n=1}^\infty \frac{\sin^6 n}{n^8} \) also converges.

Answer(s) submitted:
- B

(incorrect)
Correct Answers:
- A
6. (1 pt) For which $a$ does the following series converge?

$$\sum_{n=1}^{\infty} \frac{1}{n(n \ln n)^a}$$

Give your answer in interval notation, and type INF if you need to enter $\infty$.

**SOLUTION:**

If $a \leq 0$, then $\frac{1}{n(n \ln n)^a} \geq \frac{1}{n}$. Therefore by the Integral Test, the series converges if and only if $\int_a^{\infty} \frac{1}{x \ln x} \, dx$ converges.

Substituting $u = \ln x$, $\int_a^{\infty} \frac{1}{x \ln x} \, dx = \int_a^{\infty} \frac{1}{u} \, du$, which is divergent if and only if $a > 1$, since it is a $p$-integral.

So $\sum_{n=2}^{\infty} \frac{1}{n(n \ln n)^a}$ converges for $a \in (1, \infty)$ and diverges for $a \in (-\infty, 1]$.

**Answer(s) submitted:**

- (1, INF)
- (correct)

**Correct Answers:**

- (1, infinity)

7. (1 pt) Determine the convergence or divergence of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n n^3}{n^2 + 9}$$

- This series is absolutely convergent.
- This series is conditionally convergent.
- This series is divergent.

**SOLUTION:**

Let $a_n = \frac{(-1)^n n^3}{n^2 + 9}$. A series cannot converge if its terms do not approach 0 as $n \to \infty$. Since the degree of the numerator of $a_n$ is greater than the degree of the denominator, $\lim_{n \to \infty} a_n$ does not exist. Therefore $\sum_{n=1}^{\infty} \frac{(-1)^n n^3}{n^2 + 9}$ diverges.

**Answer(s) submitted:**

- This series is divergent.
- (correct)

**Correct Answers:**

- This series is divergent.

8. (1 pt) Determine the convergence or divergence of the series

$$\sum_{n=1}^{\infty} \frac{\sin \left( \frac{\pi n}{6} \right)}{n^3}$$

- This series is absolutely convergent.
- This series is conditionally convergent.
- This series is divergent.

**SOLUTION:**

Let $a_n = \frac{\sin \left( \frac{\pi n}{6} \right)}{n^3}$.

We have that $0 \leq |a_n| \leq \frac{1}{n^3}$. And $\sum \frac{1}{n^3}$ is convergent because it is a $p$-series with $p > 1$. So $\sum |a_n|$ is convergent by the Comparison Test. So $\sum a_n$ is absolutely convergent.

**Answer(s) submitted:**

- This series is absolutely convergent.
- (correct)

**Correct Answers:**

- This series is absolutely convergent.

9. (1 pt) Approximate the value of the series to within an error of at most $10^{-5}$.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$$

According to Equation (2):

$$|S_N - S| \leq a_{N+1}$$

what is the smallest value of $N$ that approximates $S$ to within an error of at most $10^{-5}$?

$N = \underline{\underline{18}}$.

**SOLUTION:**

Let $S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$, so that $a_n = \frac{1}{n^3}$. By Equation (2),

$$|S_N - S| \leq a_{N+1} = \frac{1}{(N+1)^3}.$$ 

To make the error less than $10^{-5}$, we must choose $N$ so that

$$\frac{1}{(N+1)^3} < 10^{-5} \quad \text{or} \quad N > 10^5 - 1 \approx 4.18.$$ 

The smallest value that satisfies the required inequality is then $N = 5$. Thus,

$$S \approx S_5 = 1 - \frac{1}{2^3} + \frac{1}{3^3} - \frac{1}{4^3} + \frac{1}{5^3} = 0.992597$$

**Answer(s) submitted:**
• 5
• $1-1/2^7+1/3^7-1/4^7+1/5^7$

(correct)

Correct Answers:
• 5
• 0.992597